HIGH-SPEED COLLISION OF SOLID BODIES WITH IDENTICAL PHYSICAL PROPERTIES

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We solve a one-dimensional problem involving the high-speed collision of a platelike projectile with a semiinfinite obstacle as target, the resulting material behavior assumed to be in the hydrodynamic regime, describable by means of a two-parameter equation of state $p = p(\rho, \varepsilon)$. The physical impact process can be divided into two stages: the first solvable analytically, the second on an electronic digital computer (the BÉSM-4), wherein we employ an artificial viscosity and a finite-difference scheme due to Richtmyer.

The collision investigated involves bodies with identical physical properties (single-phase and two-phase); a description is obtained of the basic parameters of the medium at the front of the shock wave and also throughout the whole compressed region for impact velocities ranging from 2 to 6 km/sec.

1. Statement of the Problem. We consider the impact at normal incidence of a flat plate of thickness l_0 and of diameter d_0, where $l_0/d_0 \ll 1$, into a semiinfinite target.

We assume that the initial speed of impact V_0 is such that the stresses in the deformed regions are isotropic and are equivalent to a hydrodynamic compression.

We separate the impact process into two stages. The first stage commences immediately upon impact and is characterized by the propagation into both projectile and target of planar shock waves of constant intensity.

Assuming that the pressure and mass velocity vary continuously across the contact surface ($p = p_-$, $u = u_-$), we determine, for a given impact speed V_0, from the conditions of dynamic compatibility at the front of the shock wave and the equation of state of the projectile and target material, all the parameters characterizing this stage of the impact. (Projectile parameters are indicated with a minus sign.)

We assume the first stage of the impact process to terminate at the instant t_1, when the shock wave in the projectile exits at its rear surface; at this instant the thickness of the compressed projectile is h_1, and the compressed region of the target, extending from the contact surface to the shock wave, is of depth h_2. We then have

$$t_1 = \frac{l_0}{D_- - V_0}, \qquad h_1 = \frac{l_0}{\rho/\rho_0}, \qquad h_2 = l_0 \frac{1 - h_1}{\rho/\rho_0 - 1}$$

Here D_- is the speed of the shock wave in the projectile.

Values of the parameters obtained for the first stage of the impact process serve as initial values for the second stage. The second stage of the process is essentially nonstationary and commences after reflection of the shock wave from the free surface of the projectile. This stage is characterized by the interaction of unloading waves, propagating from the free surface of the projectile, with the shock wave in the target.

As the result of this interaction, the shock wave in the target gradually decays into an elastic wave, and the impact process terminates.

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TABLE 1.

	A	b	K	E_0	γ_1	γ_2
Al	0.4827	8.0755	0.5045	−1.3241	0.2562	1.8746
Cu	0.3737	10.0094	0.3837	−1.6458	−0.1232	2.1631
Pb	0.9797	4.9428	0.9993	−2.3962	1.5188	1.2611
α-Fe	0.6765	7.0985	0.6863	−1.7741	0.7438	0.9370
γ-Fe	0.6405	7.7845	0.7215	−1.9378	0.4162	1.1672

To find the parameters which apply to the deformed regions of the impacting bodies during the second stage we apply the finite-difference scheme due to Richtmyer [1] with an artificial viscosity.

2. Equations and Boundary Conditions. In studying the second stage of the process we start from the one-dimensional hydrodynamic equations in Lagrangian variables

$$\frac{\partial R}{\partial t} = u, \quad \frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial (p+q)}{\partial r}, \quad \frac{\partial \varepsilon}{\partial t} = \frac{p+q}{\rho}\frac{\partial \rho}{\partial t},$$
$$\frac{1}{\rho} = \frac{1}{\rho_0}\frac{\partial R}{\partial t}, \quad p = p(\rho, \varepsilon) \tag{2.1}$$

Here r and t are independent Lagrangian coordinates, R is an Eulerian coordinate, ρ_0 is the material density at the instant t = 0, and ε is the specific internal energy for the deformed material.

The artificial viscosity is assumed in the form

$$q = 4a^2\rho\left(\frac{\partial u}{\partial r}\right)^2 \quad \text{for } \frac{\partial u}{\partial r} < 0, \quad q = 0 \quad \text{for } \frac{\partial u}{\partial r} \geqslant 0 \tag{2.2}$$

Here a is the coefficient of artificial viscosity, which depends on V_0 and on the metallic material $(1 < a < 2)$.

The variables appearing in the expressions in Eqs. (2.1) and (2.2), and the dimensional constants, can be put into dimensionless form by setting

$$R = l_0\bar{R}, \quad r = l_0\bar{r}, \quad t = l_0 / c_0\bar{t}, \quad \rho = \rho_0\bar{\rho}, \quad u = c_0\bar{u}, \quad \varepsilon = c_0^2\bar{\varepsilon}, \quad p = \rho_0 c_0^2\bar{p}$$

where ρ_0 and c_0 are the density and sound speed of the undeformed material. The overbars on the letters indicate nondimensional quantities; they will be omitted from now on.

The set of equations (2.1) and the expression (2.2) were written in explicit finite-difference form and solved on the BÉSM-4 with the following initial and boundary conditions.

At t = t_1 the initial conditions have the form

$$u(R, t_1) = \begin{cases} u & (0 \leqslant R \leqslant R_2) \\ 0 & (R > R_2) \end{cases}, \quad \rho(R, t_1) = \begin{cases} 1 & (R = 0) \\ \rho & (R_1 < R \leqslant R_2) \\ 1 & (R > R_2) \end{cases}$$
$$p(R, t_1) = \begin{cases} 0 & (R = 0) \\ p & (0 < R \leqslant R_2), \\ 0 & (R > R_2) \end{cases} \quad \varepsilon(R, t_1) = \begin{cases} 0 & (R = 0) \\ \varepsilon & (0 < R \leqslant R_2) \\ 0 & (R > R_2) \end{cases} \tag{2.3}$$

The boundary conditions are

$$p(0, t) = 0 \tag{2.4}$$

where R = 0 is the coordinate of the free surface of the projectile.

The coordinates R_1 and R_2 correspond, respectively, to the contact surface and to the boundary separating disturbed and undisturbed regions in the target.

3. Selection of an Equation of State. In our calculations we have used a two-parameter equation of state of the form $p = p(\rho, \varepsilon)$, given in [2] by Zharkov and Kalinin. Making use of the fact that the Grüneisen coefficient depends linearly on the specific volume, we can, after some manipulations, write the equation of state in the form

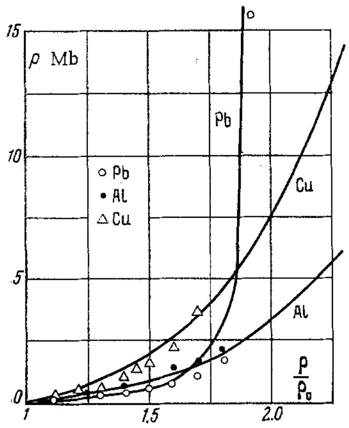

Fig. 1

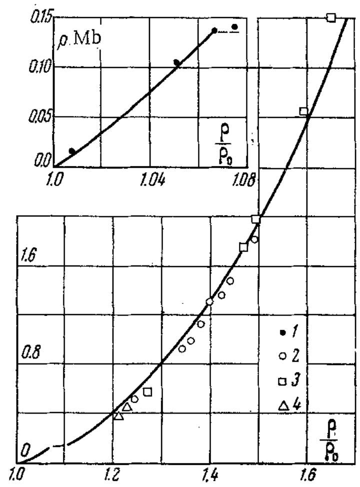

Fig. 2

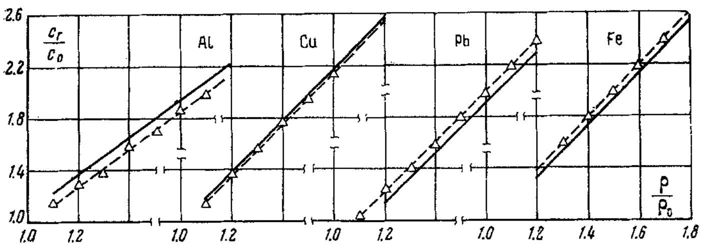

Fig. 3

$$p(\rho,\ \varepsilon) = Q\rho^{2/3} - K\rho^{4/3} + (E_0 + \varepsilon - 3Q/b + 3K\rho^{1/3})(\gamma_1\rho + \gamma_2)$$
$$Q = A\exp[b(1 - \rho^{-1/3})] \qquad (3.1)$$

Here E_0 is the specific internal energy of the body under normal conditions ($p = 0$); ε is the specific internal energy of the body in the deformed state; A, b, K, γ_1, and γ_2 are constants characterizing the material through the conditions of compressibility (dynamic, static, and mean compressibility). These constants are given in [2] for each compressibility condition, the values being dependent on the choice of a parameter m: for m = 0, 1, 2, we have the separate formulations for the Grüneisen coefficient due, respectively, to Slater, to Dugdale and MacDonald, and to Zubarev and Vashchenko.

To make use of the equation of state (3.1) for high-speed impact problems, we constructed the corresponding shock adiabats for aluminum, lead, copper, and iron.

Comparisons were made under dynamic compressibility conditions of the shock adiabat values furnished by the equation of state (3.1) with the experimental values obtained in [3-6] by Al'tshuler et al.; at high dynamic loading it was found that the best comparison was obtained when m = 0.

In Table 1 we give the values of the constants for the equation of state (3.1) for a series of metals.

The shock adiabat for iron, a material which undergoes a polymorphic transformation, was drawn (see Fig. 2) in accord with the equation of state (3.1); however, the constants appearing in this equation of state were chosen separately for the α and γ phases; the constants for the α phase were obtained from static compressibility conditions with m = 1, while those for the γ phase are those given in [7], the latter giving the best comparison with experimental values; in Fig. 2 the points 1, 2, 3, and 4 are taken from [9, 8, 3, 10], respectively.

The construction of such an adiabat for iron, one having a discontinuity but yet depending on the single equation of state (3.1), makes it possible to calculate the parameters of the collision process for iron in a continuous manner, just as would be done in the case of a collision involving single phase materials.

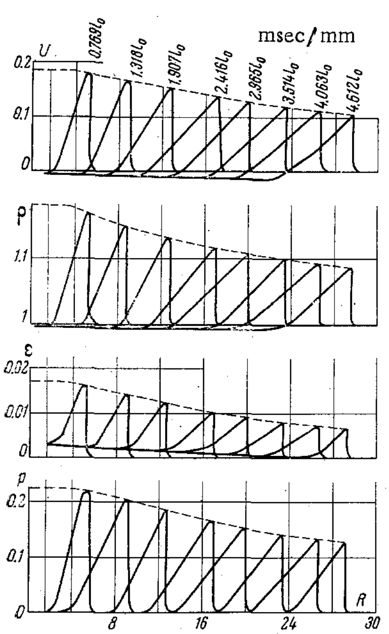

Fig. 4

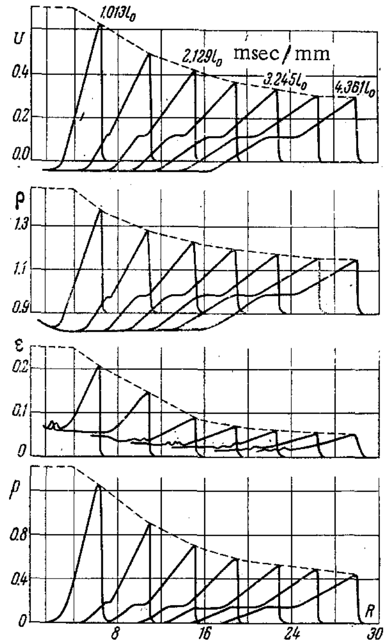

Fig. 5

The asymptote of the shock adiabat for lead is $\rho/\rho_0 = 1.92$; for copper, aluminum, and iron, $\rho/\rho_0 > 3$. This testifies to the fact that use of the equation of state (3.1) for metallic impacts may involve a wide range of impact speeds.

To estimate the equation of state (3.1) during the unloading of a shocked material, we calculated the speed of sound back of the shock front.

Noting that during unloading the motion of the medium has an adiabatic character ($\Delta S = 0$), we obtain an expression for the speed of sound in the form

$$c^2 = \frac{\partial p}{\partial \rho} + \frac{\partial p}{\partial \varepsilon}\frac{p}{\rho^2} \tag{3.2}$$

Calculations made using the formula (3.2) for values of the sound speed back of the shock front for Al, Cu, Pb, and Fe are displayed in Fig. 3 (c_0 = 5.454, 3.977, 2.256, and 4.577 km/sec, respectively). These values are in good agreement with the experimental data obtained by Al'tshuler [6] (shown by dashed curves in Fig. 3).

4. Discussion of the Results. Impact problems were solved on the BÉSM-4 for collisions of aluminum on aluminum and iron on iron for impact speeds ranging from 2 to 6 km/sec.

However, owing to limitations on the calculational mesh used, the problem of following the decay of the shock wave until it becomes an elastic wave could be handled only for small impact speeds (V_0 = 2 km/sec).

The results obtained upon solving the set of equations (2.1) with the initial and boundary conditions (2.3) and (2.4) are shown in Fig. 4 for the case involving the impact of an aluminum projectile onto an aluminum target at 2 km/sec, and in Fig. 5 for the case of iron on iron at 6 km/sec. From these graphs we can ascertain the nature of the decay of the mass velocity, density, internal energy, and pressure at the shock front and throughout the compressed region.

As the material unloads to zero initial pressure, the density of the material decreases to a value which is less than the initial normal density of the material. This demonstrates the fact that the material unloads isentropically, the isentrope being distinct from the shock adiabat.

From the graphs we can also ascertain the residual energy of the unloaded material, this being larger the larger the impact speed V_0; the value of the residual energy can be used to estimate the temperature of the material upon unloading.

In the curves for iron a rarefaction shock wave is discernable over an expanding region, a wave of this type arising in two-phase materials; the main compression shock wave in the target, however, preserves a stable configuration since the impact speed in question $V_0 > 2$ km/sec.

From the results obtained we can visualize the behavior of the free and of the contact surfaces.

In the first stage of the impact process all points, and hence those on the free and contact surfaces also, have a positive velocity; i.e., they move in the direction of the impact. In the second stage the free surface, in the process of unloading, begins to move in a direction opposite to that of the impact velocity vector, encompassing as it does so an ever larger number of points. The contact surface is also encompassed by this motion and, as unloading comes to an end, it acquires a negative velocity.

The problem described here can also be extended to the case of a target of finite thickness when appropriate criteria for the fracture of metals in unloading waves is available.

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